MATH 209: Corrections to eText

Amrit Shrestha, 2020

Section 1 – 1, page 5

EXAMPLE 3 Solving a Formula for a Particular Variable

Solve the amount formula for simple interest, $A = P + P\,r\,t$, for

(A) r in terms of the other variables

(B) P in terms of the other variables

SOLUTION

(A)

$$\begin{aligned} A &= P + \text{Prt} && \text{Reverse equation .} \\ P + \text{Prt} &= A && \text{Now isolate } r \text{ on the left sides .} \\ \text{Prt} &= A - P && \text{Divide both members by pt .} \\ r &= \frac{A - P}{\text{Pt}} \end{aligned}$$

(B)

$$\begin{aligned} A &= P + \text{Prt} && \text{Reverse equation .} \\ P + \text{Prt} &= A && \text{Factor out } P \text{ (note the use of the distributive property) .} \\ P\,(1 + \text{rt}\,) &= A && \text{Divide by } (1 + \text{rt}\,) \\ P &= \frac{A}{1 + \text{rt}} \end{aligned}$$

MATCHED PROBLEM 3

Solve $M = Nt + Nr$ for

(A) t

(B) N

)

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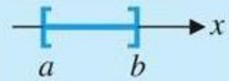
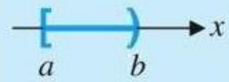
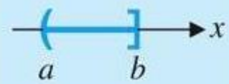
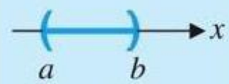
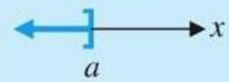
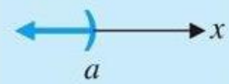
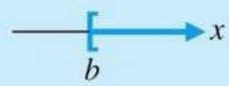
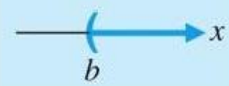
Section 1 – 1, page 7

TABLE 1

Interval Notation	Inequality Notation	Line Graph
$[a, b]$	$a \le x \le b$	
(a, b) [a, b)	$a \le x < b$	
(a, b) (a, b]	$a < x \le b$	
(a, b)	$a < x < b$	
$(-\infty, a)$ (-∞, a]	$x \le a$	
$(-\infty, a)$	$x < a$	
(b, ∞)* [b, ∞)	$x \ge b$	
(b, ∞)	$x > b$	

Section 1-1, page 11

Answers to Matched Problems

1.

$x = 4$

2.

$x = 2$

3.

(A) $t = \frac{M - Nr}{N}$

(B) $N = \frac{M}{t + r}$

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41. What can be said about the signs of the numbers a and b in each case?

(A) $ab > 0$

(B) $ab < 0$

(C) $\frac{a}{b} > 0$

(D) $\frac{a}{b} < 0$

42. What can be said about the signs of the numbers a, b, and c in each case?

(A) $abc > 0$

(B) $\frac{ab}{c} < 0$

(C) $\frac{a}{bc} > 0$

(D) $\frac{a^2}{bc} < 0$

Section 1 – 2, page 15

THEOREM 1 GRAPH OF A LINEAR EQUATION IN TWO VARIABLES

The graph of any equation of the form

$$Ax + By = C \qquad \text{(A and B not both 0)} \qquad (1)$$

is a line, and any line in a Cartesian coordinate system is the graph of an equation of this form.

If A ≠ 0 and B ≠ 0, then equation (1) can be written as

$$y = -\frac{A}{B}x + \frac{C}{B} = mx + b, \; m = 0$$

$m \neq 0$

a form we will use often. If $A = 0$ and $B \neq 0$, then equation (1) can be written as

$$y = \frac{C}{B}$$

and its graph is a **horizontal line.** If $A \neq 0$ and $B = 0$, then equation (1) can be written as

$$x = -\frac{A}{B}$$

$x = \frac{C}{A}$

Section 1 – 2, page 20

EXAMPLE 5 Using The Slope–Intercept Form

(A) Find the slope and y intercept, and graph $y = -\frac{2}{3}x - 3$.

(B) Write the equation of the line with slope $\frac{2}{3}$ and y intercept -2.

SOLUTION

(A) $\text{Slope} = m = -\frac{2}{3}$

y intercept $= b = -3$

(B) $m = \frac{2}{3}$ and $b = -2$;

thus, $y = \frac{2}{3}x - 2$

Section 1 – 2, page 21

EXAMPLE 6 Using the Point-Slope Form

(A) Find an equation for the line that has slope $\frac{1}{2}$ and passes through $(-4, 3)$. Write the final answer in the form $Ax + By = C$.

(B) Find an equation for the line that passes through the two points $(-3, 2)$ and $(-4, 5)$. Write the resulting equation in the form $y = mx + b$.

SOLUTION

(A) Use $y - y_1 = m(x - x_1)$. Let $m = \frac{1}{2}$ and $(x_1, y_1) = (-4, 3)$. Then

$$\begin{aligned} y - 3 &= \tfrac{1}{2}[x - (-4)] \\ y - 3 &= \tfrac{1}{2}(x + 4) \quad \text{Multiply by } 2. \\ 2y - 6 &= x + 4 \\ -x + 2y &= 10 \text{ or } x - 2y = -10 \end{aligned}$$

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MATCHED PROBLEM 6

(A) Find an equation for the line that has slope $\frac{2}{3}$ and passes through (6, −2). Write the resulting equation in the form $Ax + By = C$, A > 0.

(B) Find an equation for the line that passes through (2, −3) and (4, 3). Write the resulting equation in the form $y = mx + b$.

EXAMPLE 7 Cost Equation

The management of a company that manufactures roller skates has fixed costs (costs at 0 output) of $300 per day and total costs of $4,300 per day at an output of 100 pairs of skates per day. Assume that cost C is linearly related to output x.

(A) Find the slope of the line joining the points associated with outputs of 0 and 100; that is, the line passing through (0, 300) and (100, 4,300).
(B) Find an equation of the line relating output to cost. Write the final answer in the form $C = mx + b$.
(C) Graph the cost equation from part (B) for $0 \le x \le 200$.

SOLUTION

(A)
$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4{,}300 - 300}{100 - 0} \\ &= \frac{4{,}000}{100} = 10 \end{aligned}$$

REVIEW EXERCISE, page 43 – 44

13. Write the equation of a line through each indicated point with the indicated slope. Write the final answer in the form $y = mx + b$.

(A) $m = -\frac{2}{3}$; $(-3,2)$

(B) $m = 0$; (3,3)

$y = -\frac{1}{m}x + b$

32. Graph $y = mx + b$ and $y = -\frac{1}{m} + b$ simultaneously in the same coordinate system for b fixed and several different values of m, $m \neq 0$. Describe the apparent relationship between the graphs of the two equations.

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Section 2 – 1, page 48

MATCHED PROBLEM 1

Sketch the graph of each equation.

(A) $y = x^2 - 4$

(B) $y^2 = \frac{100}{x^2 + 1}$

Section 2 – 1, page 54

MATCHED PROBLEM 4

Use the functions in Example 3 to find

(A) $f(-2)$

(B) $g(-1)$

(C) $h(-8)$

(D) $\frac{f(3)}{h(5)}$

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Section 2 – 1, page 55

EXAMPLE 6 Using Function Notation

For $f(x) = x^2 - 2x + 7$, find

(A) $f(a)$
(B) $f(a + h)$
(C) $f(a + h) - f(a)$
(D) $\frac{f(a+h)-f(a)}{h}$, $h \neq 0$

SOLUTION

(A) $f(a) = a^2 - 2a + 7$
(B) $f(a + h) = (a + h)^2 - 2(a + h) + 7 = a^2 + 2ah + h^2 - 2a - 2h + 7$
(C)
$$\begin{aligned} f(a+h) - f(a) &= (a^2 + 2ah + h^2 - 2a - 2h + 7) - (a^2 - 2a + 7) \\ &= 2ah + h^2 - 2h \end{aligned}$$
(D)
$$\frac{f(a+h)-f(a)}{h} = \frac{2ah + h^2 - 2h}{h} = \boxed{\frac{h(2a + h - 2)}{h}} \quad \text{Because } h \neq 0, \frac{h}{h} = 1.$$
$$= 2a + h - 2$$

Section 2 – 1, page 57

MATCHED PROBLEM 7

The financial department in Example 6, using statistical techniques, produced the data in Table 6, where $C(x)$ is the cost in millions of dollars for manufacturing and selling x million cameras.

7

Section 2 – 1, page 61

In Problems 107–112, find and simplify each of the following.

(A) $f(x + h)$
(B) $f(x + h) - f(x)$
(C) $\frac{f(x+h)-f(x)}{h}$

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Section 2 – 2, page 64

EXAMPLE 1 Evaluating Basic Elementary Functions

Evaluate each basic elementary function at

(A) $x = 64$

(B) $x = -12.75$

Round any approximate values to four decimal places.

SOLUTION

(A)

$$
\begin{aligned}
f(64) &= 64 \\
h(64) &= 64^2 = 4{,}096 && \text{Use a calculator.} \\
m(64) &= 64^3 = 262{,}144 && \text{Use a calculator.} \\
n(64) &= \sqrt{64} = 8 \\
p(64) &= \sqrt[3]{64} = 4 \\
g(64) &= |64| = 64
\end{aligned}
$$

(B)

$$
\begin{aligned}
f(-12.75) &= -12.75 \\
h(-12.75) &= (-12.75)^2 = 162.5625 && \text{Use a calculator.} \\
m(-12.75) &= (-12.75)^3 \approx -2{,}072.6719 && \text{Use a calculator.} \\
n(-12.75) &= \sqrt{-12.75} && \text{Not a real number.} \\
p(-12.75) &= \sqrt[3]{-12.75} \approx -2.3362 && \text{Use a calculator.} \\
g(-12.75) &= |-12.75| = 12.75
\end{aligned}
$$

Section 2 – 2, page 66

MATCHED PROBLEM 2

(A) How are the graphs of $y = \sqrt{x} + 5$ and $y = \sqrt{x} - 4$ related to the graph of $y = \sqrt{x}$? Confirm your answer by graphing all three functions simultaneously in the same coordinate system.

(B) How are the graphs of $y = \sqrt{x+5}$ and $y = \sqrt{x-4}$ related to the graph of $y = \sqrt{x}$? Confirm your answer by graphing all three functions simultaneously in the same coordinate system.

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Section 2 – 2, page 68

Explore & Discuss 2

(A) Graph $y = Ax^2$ for $A = 1, 4$, and $\frac{1}{4}$ simultaneously in the same coordinate system.

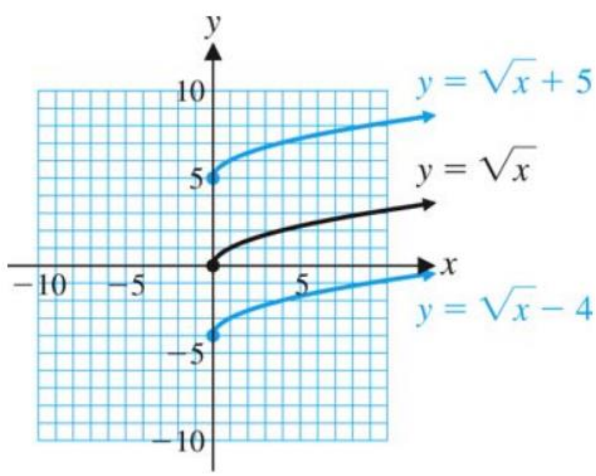
(B) Graph $y = Ax^2$ for $A = -1, -4$, and $-\frac{1}{4}$ simultaneously in the same coordinate system.

(C) Describe the relationship between the graph of $h(x) = x^2$ and the graph of $G(x) = Ax^2$ for A any real number.

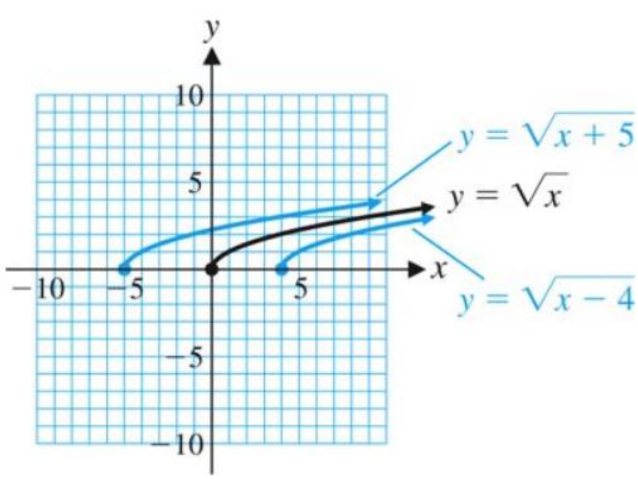
Section 2-2, p.71

2.

(A) The graph of $y = \sqrt{x} + 5$ is the same as the graph of $y = \sqrt{x}$. shifted upward 5 units, and the graph of $y = \sqrt{x} - 4$ is the same as the graph of $y = \sqrt{x}$. shifted downward 4 units. The figure confirms these conclusions.



(B) The graph of $y = \sqrt{x+5}$ is the same as the graph of $y = \sqrt{x}$ shifted to the left 5 units, and the graph of $y = \sqrt{x-4}$ is the same as the graph of $y = \sqrt{x}$ shifted to the right 4 units. The figure confirms these conclusions.



Section 2 – 3, page 90

Exercise 2-3

A

vertex form

In Problems 1–4, complete the square and find the standard form of each quadratic function.

1. $f(x) = x^2 - 4x + 3$

2. $g(x) = x^2 - 2x - 5$

3. $m(x) = -x^2 + 6x - 4$

4. $n(x) = -x^2 + 8x - 9$

Solution Manual, page S-48

1.

$$\begin{aligned} f(x) &= x^2 - 4x + 3 \\ &= \left(x^2 - 4x\right) + 3 \\ &= \left(x^2 - 4x + 4\right) + 3 - 4 \quad \text{(completing the square)} \\ &= (x - 2)^2 - 1 \quad \text{(}\overset{\text{vertex}}{\cancel{\text{standard}}}\text{ form)} \end{aligned}$$

3.

$$\begin{aligned} f(x) &= -x^2 + 6x - 4 \\ &= -\left(x^2 - 6x\right) - 4 \\ &= -\left(x^2 - 6x + 9\right) - 4 + 9 \quad \text{(completing the square)} \\ &= -(x - 3)^2 + 5 \quad \text{(}\overset{\text{vertex}}{\cancel{\text{standard}}}\text{ form)} \end{aligned}$$

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Solution to Exercise 2-4 #63, page S-61

63. *Finance.* A person wishes to have $15,000 cash for a new car 5 years from now. How much should be placed in an account now, if the account pays 6.75% compounded weekly? Compute the answer to the nearest dollar.

63.

With $P = 7{,}500$ and $r = 0.0835$, we have:

$A = 7{,}500\ e^{0.0835t}$

(A) $A = 7{,}500e^{(0.0835)5.5} = 7{,}500\ e^{0.45925} \approx 11{,}871.65$ Thus, there will be $11,871.65 in the account after 5.5 years.

(B) $A = 7{,}500e^{(0.0835)12} = 7{,}500\ e^{1.002} \approx 20{,}427.93$ Thus, there will be $20,427.93 in the account after 12 years.

Correct Solution:

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

$$15000 = P\left(1 + \frac{.0675}{52}\right)^{52x5}$$

P = $10,705.62

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Solve for N when $t = 0.6$:

(A) $N = 25\ e^{1.386\,(0.6)}$ Use a calculator .

$= 57$ bacteria

Solve for N when $t = 3.5$:

(B) $N = 25\ e^{1.386\,(3.5)}$ Use a calculator .

$= 3{,}197$ bacteria

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(A)

Solve for A when $t = 15{,}000$:

$A = 500\, e^{-0.000124(15{,}000)}$ Use a calculator .

$= 77.84$ milligrams

(B)

Solve for A when $t = 45{,}000$:

$A = 500\, e^{-0.000124(45{,}000)}$ Use a calculator .

$= 1.89$ milligrams

Section 2 – 4, page 102

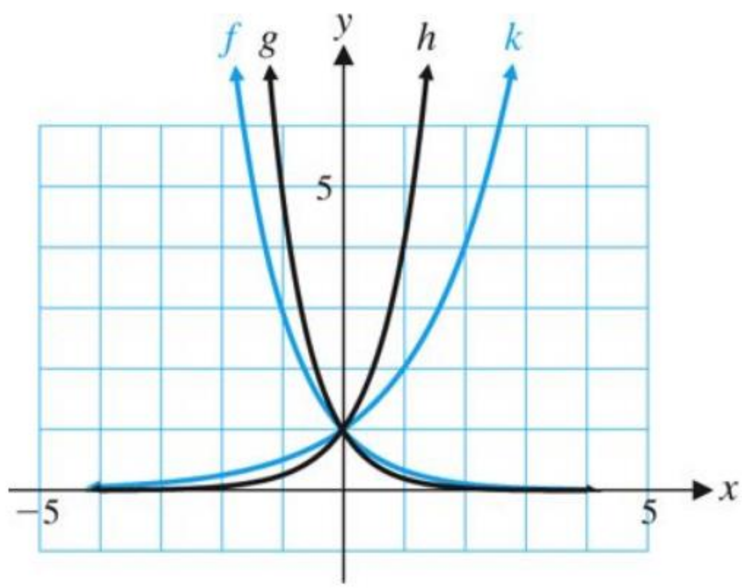
1. Match each equation with the graph of f, g, h, or k in the figure.

(A) $y = 2^x$

(B) $y = (0.2)^x$

(C) $y = 4^x$

(D) $y = \left(\frac{1}{3}\right)^x$



2. Match each equation with the graph of f, g, h, or k in the figure.

(A) $y = \left(\frac{1}{4}\right)^x$

(B) $y = (0.5)^x$

(C) $y = 5^x$

(D) $y = 3^x$

Section 2 – 5, page 108

EXAMPLE 1 Logarithmic-Exponential Conversions

Change each logarithmic form to an equivalent exponential form:

(A) $\log_5 25 = 2$

(B) $\log_9 3 = \frac{1}{2}$

(C) $\log_2 \left(\frac{1}{4}\right) = -2$

SOLUTION

(A) $\log_5 25 = 2$ is equivalent to $25 = 5^2$

(B) $\log_9 3 = \frac{1}{2}$ is equivalent to $3 = 9^{1/2}$

(C) $\log_2 \left(\frac{1}{4}\right) = -2$ is equivalent to $\frac{1}{4} = 2^{-2}$

MATCHED PROBLEM 1

Change each logarithmic form to an equivalent exponential form:

(A) $\log_3 9 = 2$

(B) $\log_4 2 = \frac{1}{2}$

(C) $\log_3 \left(\frac{1}{9}\right) = -2$

EXAMPLE 2 Exponential-Logarithmic Conversions

Change each exponential form to an equivalent logarithmic form:

(A) $64 = 4^3$

(B) $6 = \sqrt{36}$

(C) $\frac{1}{8} = 2^{-3}$

SOLUTION

(A) $64 = 4^3$ is equivalent to $\log_4 64 = 3$

(B) $6 = \sqrt{36}$ is is equivalent to $\log_{36} 6 = \frac{1}{2}$

(C) $\frac{1}{8} = 2^{-3}$ is equivalent to $\log_2 \left(\frac{1}{8}\right) = -3$

Section 2 – 5, page 109

MATCHED PROBLEM 2

Change each exponential form to an equivalent logarithmic form:

(A) $49 = 7^2$

(B) $3 = \sqrt{9}$

(C) $\frac{1}{3} = 3^{-1}$

Section 2 – 5, page 110

EXAMPLE 4 Using Logarithmic Properties

(A)

$$\log_b \frac{wx}{yz} \boxed{\begin{aligned} &= \log_b wx - \log_b yz \\ &= \log_b w + \log_b x - (\log_b y + \log_b z) \end{aligned}} = \log_b w + \log_b x - \log_b y - \log_b z$$

(B)

$$\log_b (wx)^{3/5} \boxed{= \tfrac{3}{5} \log_b wx} = \tfrac{3}{5}(\log_b w + \log_b x)$$

(C) $e^{x \log_e b} = e^{\log_e b^x} = b^x$

(D) $\dfrac{\log_e x}{\log_e b} = \dfrac{\log_e \left(b^{\log_b x} \right)}{\log_e b} = \dfrac{(\log_b x)(\log_e b)}{\log_e b} = \log_b x$

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Section 2 – 5, page 110

MATCHED PROBLEM 4

Write in simpler forms, as in Example 4.

(A) $\log_b \frac{R}{ST}$

(B) $\log_b \left(\frac{R}{S} \right)^{2/3}$

(C) $2^{u \log_2 b}$

(D) $\frac{\log_2 x}{\log_2 b}$

Section 2 – 5, page 111

Explore & Discuss 2

Discuss the relationship between each of the following pairs of expressions. If the two expressions are equivalent, explain why. If they are not, give an example.

(A) $\log_b M - \log_b N$; $\frac{\log_b M}{\log_b N}$

(B) $\log_b M - \log_b N$; $\log_b \frac{M}{N}$

(C) $\log_b M + \log_b N$; $\log_b MN$

(D) $\log_b M + \log_b N$; $\log_b (M + N)$

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Section 2 – 5, page 112

EXAMPLE 8 Solving $\log_b x = y$ for x

Find x to four decimal places, given the indicated logarithm:

(A) $\log x = -2.315$
(B) $\ln x = 2.386$

SOLUTION

(A)

$\log x =$	-2.315	Change to equivalent exponential form.
$x =$	$10^{-2.315}$	Evaluate with a calculator.
$=$	0.0048	

(B)

$\ln x =$	2.386	Change to equivalent exponential form.
$x =$	$e^{2.386}$	Evaluate with a calculator.
$=$	10.8699	

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SOLUTION

(A)

$10^x =$	2	Take common logarithms of both sides.
$\log 10^x =$	$\log 2$	Property 3
$x =$	$\log 2$	Use a calculator.
$=$	0.3010	To four decimal places

(B)

$e^x =$	3	Take natural logarithms of both sides.
$\ln e^x =$	$\ln 3$	Property 3
$x =$	$\ln 3$	Use a calculator.
$=$	1.0986	To four decimal places

(C)

$3^x =$	4	Take either natural or common logarithms of both sides. (We choose common logarithms.)
$\log 3^x =$	$\log 4$	Property 7
$x \log 3 =$	$\log 4$	Solve for x.
$x =$	$\frac{\log 4}{\log 3}$	Use a calculator.
$=$	1.2619	To four decimal places

REMARK

In the usual notation for natural logarithms, the simplifications of **Example 4**, parts (C) and (D) on page 114, become

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$$e^{x \ln b} = b^x \quad \text{and} \quad \frac{\ln x}{\ln b} = \log_b x$$

With these formulas we can change an exponential function with base b, or a logarithmic function with base b, to expressions involving exponential or logarithmic functions, respectively, to the base e. Such **change-of-base formulas** are useful in calculus.

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Section 2 – 5, page 116

Answers to Matched Problems

1.

(A) $9 = 3^2$

(B) $2 = 4^{1/2}$

(C) $\frac{1}{9} = 3^{-2}$

2.

(A) $\log_7 49 = 2$

(B) $\log_9 3 = \frac{1}{2}$

(C) $\log_3 \left(\frac{1}{3} \right) = -1$

3.

(A) $y = \frac{3}{2}$

(B) $x = \frac{1}{3}$

(C) $b = 10$

4.

(A) $\log_b R - \log_b S - \log_b T$

(B) $\frac{2}{3} (\log_b R - \log_b S)$

(C) b^u

(D) $\log_b x$

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Section 2 – 5, page 121

5. For $f(x) = 2x - 1$ and $g(x) = x^2 - 2x$, find:

(A) $f(-2) + g(-1)$

(B) $f(0) \cdot g(4)$

(C) $\frac{g(2)}{f(3)}$

(D) $\frac{f(3)}{g(2)}$

Section 2 – 5, page 122

48. Find the domain of each function:

(A) $f(x) = \frac{2x-5}{x^2-x-}$

(B) $g(x) = \frac{3x}{\sqrt{5-x}}$

58. Let $f(x) = 3 - 2x$. Find

(A) $f(2)$

(B) $f(2 + h)$

(C) $f(2 + h) - f(2)$

(D) $\frac{f(2+h)-f(2)}{h}$

---

59. Let $f(x) = x^2 - 3x + 1$. Find

(A) $f(a)$

(B) $f(a + h)$

(C) $f(a + h) - f(a)$

(D) $\frac{f(a+h)-f(a)}{h}$

29¢

Section 3 – 1, page 135

43. A radio commercial for a loan company states: "You only pay 29$ a day for each $500 borrowed." If you borrow $1,500 for 120 days, what amount will you repay, and what annual interest rate is the company actually charging?

Section 3 – 2, page 146

EXAMPLE 6 Using APY to Compare Investments

Find the APYs (expressed as a percentage, correct to three decimal places) for each of the banks in **Table 1** and compare these CDs.

SOLUTION

$$\begin{aligned}
\text{Advanta: APY} &= \left(1+\frac{0.0493}{12}\right)^{12} - 1 = 0.05043 \text{ or } 5.043\% \\
\text{DeepGreen: APY} &= \left(1+\frac{0.0493}{12}\right)^{365} - 1 = 0.05074 \text{ or } 5.074\% \\
\text{Charter One: APY} &= \left(1+\frac{0.0497}{4}\right)^{4} - 1 = 0.05063 \text{ or } 5.063\% \\
\text{Liberty: APY} &= e^{0.0494} - 1 = 0.05064 \text{ or } 5.064\%
\end{aligned}$$

365

Section 3 – 4, Solution to MATCHED PROBLEM 6, page 168

6.

Choose the 0% financing.

PMT with 5.7% compounded monthly will be $518.32, which is less than monthly payment of $525 with the 0% financing.

You should still choose the $3,000 rebate.

Section 4 – 1, page 179

EXAMPLE 2 Solving a System by Graphing

Solve each of the following systems by graphing:

(A)
$$\begin{aligned} x - 2y &= 2 \\ x + y &= 5 \end{aligned}$$

(B)
$$\begin{aligned} x + 2y &= -4 \\ 2x + 4y &= 8 \end{aligned}$$

(C)
$$\begin{aligned} 2x + 4y &= 8 \\ x + 2y &= 4 \end{aligned}$$

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Section 4 – 1, page 180

MATCHED PROBLEM 2

Solve each of the following systems by graphing:

(A)
$$\begin{aligned} x + y &= 4 \\ 2x - y &= 2 \end{aligned}$$

(B)
$$\begin{aligned} 6x - 3y &= 9 \\ 2x - y &= 3 \end{aligned}$$

(C)
$$\begin{aligned} 2x - y &= 4 \\ 6x - 3y &= -18 \end{aligned}$$

We now define some terms that we can use to describe the different types of solutions to systems of equations that we will encounter.

Section 4 – 1, page 188

49. The coefficients of the three systems given below are very similar. One might guess that the solution sets to the three systems would also be nearly identical. Develop evidence for or against this guess by considering graphs of the systems and solutions obtained using substitution or elimination by addition.

(A)
$$\begin{aligned} 5x + 4y &= 4 \\ 11x + 9y &= 4 \end{aligned}$$

(B)
$$\begin{aligned} 5x + 4y &= 4 \\ 11x + 8y &= 4 \end{aligned}$$

(C)
$$\begin{aligned} 5x + 4y &= 4 \\ 10x + 8y &= 4 \end{aligned}$$

50. Repeat Problem 49 for the following systems:

(A)
$$\begin{aligned} 6x - 5y &= 10 \\ -13x + 11y &= -20 \end{aligned}$$

(B)
$$\begin{aligned} 6x - 5y &= 10 \\ -13x + 10y &= -20 \end{aligned}$$

(C)
$$\begin{aligned} 6x - 5y &= 10 \\ -12x + 10y &= -20 \end{aligned}$$

Section 4 – 3, page 201

EXAMPLE 1 Reduced Forms

The following matrices are not in reduced form. Indicate which condition in the definition is violated for each matrix. State the row operation(s) required to transform the matrix into reduced form and find the reduced form.

(A) $\left[\begin{array}{cc|c} 0 & 1 & -2 \\ 1 & 0 & 3 \end{array}\right]$

(B) $\left[\begin{array}{ccc|c} 1 & 2 & -2 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right]$

(C) $\left[\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 0 & 0 \\ 0 & 1 & -2 \end{array}\right]$

(D) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 1 & -5 \end{array}\right]$

MATCHED PROBLEM 1

The matrices below are not in reduced form. Indicate which condition in the definition is violated for each matrix. State the row operation(s) required to transform the matrix into reduced form and find the reduced form.

(A) $\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 3 & -6 \end{array}\right]$

(B) $\left[\begin{array}{ccc|c} 1 & 5 & 4 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(C) $\left[\begin{array}{ccc|c} 0 & 1 & 0 & -3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]$

(D) $\left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{array}\right]$

Section 4 – 3, page 209 (Answer to Matched Problem 5)

5.

$\boxed{x_1 = s + 7,\ x_2 = s,\ x_3 = t - 2,\ x_4 = -3t - 1,\ x_5 = t}$, s and t any real numbers

$$\begin{cases} x_1 = .5 - .5s + 1.5t \\ x_2 = -5 - 6s + 6t \\ x_3 = -1.5 - 2.5s + 2.5t \\ x_4 = s,\ x_5 = t \end{cases}$$

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Section 4 – 4, page 216

SOLUTION

(A) $$A + B = \begin{array}{cc} \text{Compact} & \text{Luxury} \end{array}$$
$$\begin{array}{c} \\ \\ \end{array}\begin{bmatrix} \$258{,}200 & \$456{,}000 \\ \$430{,}000 & \$322{,}00 \end{bmatrix} \begin{array}{l} \text{Ms. Smith} \\ \text{Mr.Jones} \end{array}$$

(B) $$B - A = \begin{array}{cc} \text{Compact} & \text{Luxury} \end{array}$$
$$\begin{bmatrix} \$174{,}000 & \$280{,}000 \\ \$178{,}000 & \$322{,}00 \end{bmatrix} \begin{array}{l} \text{Ms. Smith} \\ \text{Mr.Jones} \end{array}$$

(C) $$0.05\,B = \begin{bmatrix} (0.05)(\$228{,}000) & (0.05)(\$368{,}000) \\ (0.05)(\$304{,}000) & (0.05)(\$322{,}000) \end{bmatrix}$$
$$= \begin{bmatrix} \$11{,}400 & \$18{,}400 \\ \$15{,}200 & \$16{,}100 \end{bmatrix} \begin{array}{l} \text{Ms. Smith} \\ \text{Mr. Jones} \end{array}$$

Section 4 – 4, page 219

MATCHED PROBLEM 8

Find each product, if it is defined:

(A) $$\begin{bmatrix} -1 & 0 & 3 & -2 \\ 1 & 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 2 & 3 \\ 1 & 0 \end{bmatrix}$$

(B) $$\begin{bmatrix} -1 & 1 \\ 2 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 0 & 3 & -2 \\ 1 & 2 & 2 & 0 \end{bmatrix}$$

(C) $$\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix}$$

(D) $$\begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

(E) $$\begin{bmatrix} 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$$

(F) $$\begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & -2 & 1 \end{bmatrix}$$

Section 4 – 4, page 222

5.

(A) $\begin{bmatrix} \$\,235{,}000 & \$\,422{,}000 \\ \$\,372{,}000 & \$\,298{,}000 \end{bmatrix}$

(B) $\begin{bmatrix} \$\,145{,}000 & \$\,268{,}000 \\ \$\,160{,}000 & \$\,254{,}000 \end{bmatrix}$

(C) $\begin{bmatrix} \$\,9{,}500 & \$\,17{,}250 \\ \$\,13{,}300 & \$\,13{,}800 \end{bmatrix}$

6.

[8]

7.

$$\begin{bmatrix} 5 & 1.5 \end{bmatrix} \begin{bmatrix} 9 \\ 6 \end{bmatrix} = \begin{bmatrix} 54 \end{bmatrix}, \quad \text{or } \$54$$

8.

(A) Not defined

(B) $\begin{bmatrix} 2 & 2 & -1 & 2 \\ 1 & 6 & 12 & -4 \\ -1 & 0 & 3 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} -6 & -12 \\ 3 & 6 \end{bmatrix}$

(E) [11]

(F) $\begin{bmatrix} 12 & -8 & 4 \\ 6 & -4 & 2 \\ 9 & -6 & 3 \end{bmatrix}$

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Section 4 – 5, page 228

EXAMPLE 1 Identity Matrix Multiplication

(A) $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -2 & 5 \\ 0 & 2 & -3 \\ -1 & 4 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 5 \\ 0 & 2 & -3 \\ -1 & 4 & -2 \end{bmatrix}$$

(B) $$\begin{bmatrix} 3 & -2 & 5 \\ 0 & 2 & -3 \\ -1 & 4 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 5 \\ 0 & 2 & -3 \\ -1 & 4 & -2 \end{bmatrix}$$

(C) $$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix}$$

(D) $$\begin{bmatrix} 2 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix}$$

MATCHED PROBLEM 1

Multiply:

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 5 & 7 \end{bmatrix}$ and $\begin{bmatrix} 2 & -3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 4 & 2 \\ 3 & -5 \\ 6 & 8 \end{bmatrix}$ and $\begin{bmatrix} 4 & 2 \\ 3 & -5 \\ 6 & 8 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Explore & Discuss 1

The only real number solutions to the equation $x^2 = 1$ are $x = 1$ and $x = -1$.

(A) Show that $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ satisfies $A^2 = I$, where I is the 2×2 identity.

(B) Show that $B = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ satisfies $B^2 = I$.

(C) Find a 2×2 matrix with all elements nonzero whose square is the 2×2 identity matrix.

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Section 4 – 5, page 236

Answers to Matched Problems

1.

(A) $\begin{bmatrix} 2 & -3 \\ 5 & 7 \end{bmatrix}$

(B) $\begin{bmatrix} 4 & 2 \\ 3 & -5 \\ 6 & 8 \end{bmatrix}$

2.

(A) $\left[\begin{array}{ccc|ccc} 3 & -1 & 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

(B) $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & -1 & -1 & 2 \end{array}\right]$

(C) $\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Exercise 4-5, #29 – 38, page 237

B

$MM^{-1} = I$

Given M in Problems 29–38, find M^{-1} and show that $M^{-1} = I$.

Section 4 – 6, page 240

EXAMPLE 3 Using Inverses to Solve Systems of Equations

Use matrix inverse methods to solve each of the following systems:

(A)
$$\begin{aligned} x_1 - x_2 + x_3 &= 3 \\ 2x_2 - x_3 &= 1 \\ 2x_1 + 3x_2 &= 4 \end{aligned}$$

(B)
$$\begin{aligned} x_1 - x_2 + x_3 &= -5 \\ 2x_2 - x_3 &= 2 \\ 2x_1 + 3x_2 &= -3 \end{aligned}$$

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MATCHED PROBLEM 3

Use matrix inverse methods to solve each of the following systems (see Matched Problem 2):

(A)
$$\begin{aligned} 3x_1 - x_2 + x_3 &= 3 \\ -x_1 + x_2 \quad &= -3 \\ x_1 + \quad x_3 &= 2 \end{aligned}$$

(B)
$$\begin{aligned} 3x_1 - x_2 + x_3 &= -5 \\ -x_1 + x_2 \quad &= 1 \\ x_1 + \quad x_3 &= -4 \end{aligned}$$

REVIEW EXERCISE, page 257

26. Solve by Gauss-Jordan elimination:

(A)
$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 1 \\ 2x_1 + 3x_2 + 4x_3 &= 3 \\ x_1 + 2x_2 + \quad x_3 &= 3 \end{aligned}$$

(B)
$$\begin{aligned} x_1 + 2x_2 - x_3 &= 2 \\ 2x_1 + 3x_2 + x_3 &= -3 \\ 3x_1 + 5x_2 \quad &= -1 \end{aligned}$$

(C)
$$\begin{aligned} x_1 + x_2 + x_3 &= 8 \\ 3x_1 + 2x_2 + 4x_3 &= 21 \end{aligned}$$

34. Solve the system

0.04

$$\begin{aligned} 0.4x_1 + 0.05x_2 + 0.06x_3 &= 360 \\ 0.04x_1 + 0.05x_2 - 0.04x_3 &= 120 \\ x_1 + x_2 + x_3 &= 7{,}000 \end{aligned}$$

by writing it as a matrix equation and using the inverse of the coefficient matrix. (Before starting, multiply the first two equations by 100 to eliminate decimals. Also, see Problem 33.)

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Section 5 – 2, page 274

3.

(A)

	Labor-Hours Required		Maximum Labor-Hours Available per Month
	Two-Person Boat	Four-Person Boat	
Cutting Department	0.9	1.8	864
Assembly Department	0.8	1.2	672

(B)

$$\begin{aligned} 0.9x + 1.8y &\le 864 \\ 0.8x + 1.2y &\le 672 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$

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Answers to Matched Problems

1.

(A)

x = number of two-person boats produced each month
y = number of four-person boats produced each month

(B)

	Labor-Hours Required		Maximum Labor-Hours Available per Month
	Two-Person Boat	Four-Person Boat	
Cutting department	0.9	1.8	864
Assembly department	0.8	1.2	672
Profit per boat	\$25	\$40	

(C) $P = 25x + 40y$

(D)

$$\begin{aligned} 0.9x + 1.8y &\le 864 \\ 0.8x + 1.2y &\le 672 \\ x, y &\ge 0 \end{aligned}$$

Section 6 – 3, page 322

EXAMPLE 2 Solving a Minimization Problem

Solve the following minimization problem by maximizing the dual:

$$\begin{array}{ll} \text{Minimize} & C = 40x_1 + 12x_2 + 40x_3 \\ \text{subject to} & 2x_1 + x_2 + 5x_3 \geq 20 \\ & 4x_1 + x_1 + x_2 \geq 30 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

SOLUTION

From **Example 1** the dual is

Maximize

$$\begin{array}{ll} \text{Minimize} & P = 20y_1 + 30y_2 \\ \text{subject to} & 2y_1 + 4y_2 \leq 20 \quad 40 \\ & y_1 + y_2 \leq 12 \\ & 5y_1 + y_2 \leq 40 \\ & y_1, y_2 \geq 0 \end{array}$$

Section 6 – 4, page 345

15. $\boxed{\text{Maximize}}$ subject to

$$\begin{aligned} C &= -5x_1 - 12x_2 + 16x_3 \\ x_1 + 2x_2 + x_3 &\leq 10 \\ 2x_1 + 3x_2 + x_3 &\geq 6 \\ 2x_1 + x_2 - x_3 &= 1 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Minimize

Section 9 – 2, page 366

EXAMPLE 1 Recognizing Regular Matrices

Which of the following matrices are regular?

(A) $P = \begin{bmatrix} .8 & .2 \\ .6 & .4 \end{bmatrix}$

(B) $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $P = \begin{bmatrix} .5 & .5 & 0 \\ 0 & .5 & .5 \\ 1 & 0 & 0 \end{bmatrix}$

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MATCHED PROBLEM 1

Which of the following matrices are regular?

(A) $P = \begin{bmatrix} .3 & .7 \\ 1 & 0 \end{bmatrix}$

(B) $P = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

(C) $P = \begin{bmatrix} 0 & 1 & 0 \\ .5 & 0 & .5 \\ .5 & 0 & .5 \end{bmatrix}$

THEOREM 1 PROPERTIES OF REGULAR MARKOV CHAINS

Let P be the transition matrix for a regular Markov chain.

(A) There is a unique stationary matrix S that can be found by solving the equation

$$SP = S$$

(B) Given any initial-state matrix S_0, the state matrices S_k approach the stationary matrix S.

) The matrices P^k approach a **limiting matrix** $\bar{P}$, where each row of $\bar{P}$ is equal to the

Section 9 – 3, page 375

EXAMPLE 1 Recognizing Absorbing States

Identify any absorbing states for the following transition matrices:

(A) $P = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ .5 & .5 & 0 \\ 0 & .5 & .5 \end{bmatrix} \end{array}$

(B) $P = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$

MATCHED PROBLEM 1

Identify any absorbing states for the following transition matrices:

(A) $P = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} .5 & 0 & .5 \\ 0 & 1 & 0 \\ 0 & .5 & .5 \end{bmatrix} \end{array}$

(B) $P = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$

Section 9 – 3, page 376

EXAMPLE 2 Recogning Absorbing Markov Chains

Use a transition diagram to determine whether P is the transition matrix for an absorbing Markov chain.

(A) $$P = \begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ .5 & .5 & 0 \\ 0 & .5 & .5 \end{bmatrix} \end{array}$$

(B) $$P = \begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

Section 9 – 3, page 377

MATCHED PROBLEM 2

Use a transition diagram to determine whether P is the transition matrix for an absorbing Markov chain.

(A) $$P = \begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} .5 & 0 & .5 \\ 0 & 1 & 0 \\ 0 & .5 & .5 \end{bmatrix} \end{array}$$

(B) $$P = \begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Section 9 – 3, page 380-381

EXAMPLE 4 Finding the Limiting Matrix

(A) Find the limiting matrix $\bar{P}$ for the standard form P found in Example 3.

(B) Use $\bar{P}$ to find the limit of the successive state matrices for $S_0 = [\,0 \quad 0 \quad 1\,]$.

(C) Use $\bar{P}$ to find the limit of the successive state matrices for $S_0 = [\,.5 \quad 0 \quad .5\,]$.

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THEOREM 3 PROPERTIES OF THE LIMITING MATRIX $\overline{P}$

If P is a transition matrix in standard form for an absorbing Markov chain, F is the fundamental matrix, and $\overline{P}$ is the limiting matrix, then

(A) The entry in row i and column j of $\overline{P}$ is the long-run probability of going from state i to state j. For the nonabsorbing states, these probabilities are also the entries in the matrix FR used to form $\overline{P}$.

(B) The sum of the entries in each row of the fundamental matrix F is the average number of trials it will take to go from each nonabsorbing state to some absorbing state.

(Note that the rows of both F and FR correspond to the nonabsorbing states in the order given in the standard form P.)

INSIGHT

1. The zero matrix in the lower right corner of the limiting matrix $\overline{P}$ in **Theorem 2** indicates that the long-run probability of going from any nonabsorbing state to any other nonabsorbing state is always 0. That is, in the long run, all elements in an absorbing Markov chain end up in one of the absorbing states.

2. If the transition matrix for an absorbing Markov chain is not a standard form, it is still possible to find a limiting matrix (see Problems 37 and 38, Exercise 9-3). However, it is customary to use a standard form when investigating the limiting behavior of an absorbing chain.

Section 9 – 3, page 384 – 385

Answers to Matched Problems

1.

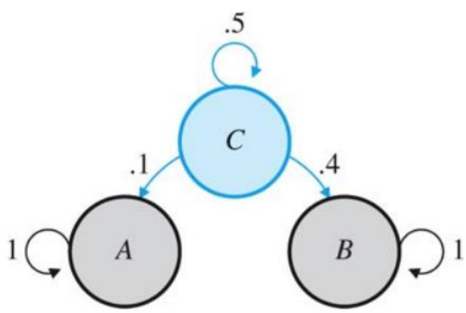
(A) State B is absorbing.
(B) State C is absorbing.

2.

(A) Absorbing Markov chain
(B) Not an absorbing Markov chain

3.

(A)



(B) $$P = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ .1 & .4 & .5 \end{bmatrix} \end{array}$$

(C) Company A will purchase 20% of the farms and company B will purchase 80%.
(D) Company A will purchase 60% of the farms and company B will purchase 40%.

4.

(A) $$\bar{P} = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ .2 & .8 & 0 \end{bmatrix} \end{array}$$

(B) [.2 .8 0]
(C) [.6 .4 0]

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35. For matrix P from Problem 29 with

(A) $S_0 = [\,0 \quad 0 \quad 1\,]$

(B) $S_0 = [\,.2 \quad .5 \quad .3\,]$

36. For matrix P from Problem 30 with

(A) $S_0 = [\,0 \quad 0 \quad 1\,]$

(B) $S_0 = [\,.2 \quad .5 \quad .3\,]$

37. For matrix P from Problem 31 with

(A) $S_0 = [\ 0 \quad 0 \quad 1\ ]$

(B) $S_0 = [\ .2 \quad .5 \quad .3\ ]$

38. For matrix P from Problem 32 with

(A) $S_0 = [\ 0 \quad 0 \quad 1\ ]$

(B) $S_0 = [\ .2 \quad .5 \quad .3\ ]$

39. For matrix P from Problem 33 with

(A) $S_0 = [\ 0 \quad 0 \quad 0 \quad 1\ ]$

(B) $S_0 = [\ 0 \quad 0 \quad .4 \quad .6\ ]$

(C) $S_0 = [\ 0 \quad 0 \quad .4 \quad .6\ ]$

(D) $S_0 = [\ .1 \quad .2 \quad .3 \quad .4\ ]$

40. For matrix P from Problem 34 with

(A) $S_0 = [\ 0 \quad 0 \quad 0 \quad 1\ ]$

(B) $S_0 = [\ 0 \quad 0 \quad .4 \quad .6\ ]$

(C) $S_0 = [\ 0 \quad 0 \quad .4 \quad .6\ ]$

(D) $S_0 = [\ .1 \quad .2 \quad .3 \quad .4\ ]$

Solution to Exercise 9-3 #57, page 388 and S-395

57. *Marketing.* Three electronics firms are aggressively marketing their graphing calculators to high school and college mathematics departments by offering volume discounts, complimentary display equipment, and assistance with curriculum development. Due to the amount of equipment involved and the necessary curriculum changes, once a department decides to use a particular calculator in their courses, they never switch to another brand or stop using calculators. Each year, 6% of the departments decide to use calculators from company *A*, 3% decide to use calculators from company *B*, 11% decide to use calculators from company *C*, and the remainder decide not to use any calculators in their courses.

(A) In the long run, what is the market share of each company?

(B) On the average, how many years will it take a department to decide to use calculators from one of these companies in their courses?

57. A transition matrix in standard form for this problem is:

$$P = \begin{array}{c} \\ A \\ B \\ C \\ N \end{array} \!\!\begin{array}{c} \begin{array}{cccc} A & B & C & N \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \overset{.06}{.6} & \overset{.03}{.3} & .11 & .8 \end{bmatrix} \end{array}$$

For this matrix, we have $R = [.6\ .3\ .11]$ and $Q = [.8]$. The limiting matrix for P has the form:

$$\bar{P} = \left[\begin{array}{c|c} I & 0 \\ \hline FR & 0 \end{array}\right]$$

where $F = (I - Q)-1 = ([1] - [.8])-1 = [.2]-1 = 5$ Now, $FR = [5][.6\ .3\ .11] = [.3\ .15\ .55]$ and

$$\bar{P} = \begin{array}{c} \\ A \\ B \\ C \\ N \end{array} \!\!\begin{array}{c} \begin{array}{cccc} A & B & C & N \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ .3 & .15 & .55 & 0 \end{bmatrix} \end{array}$$

(A) In the long run, the market share of each company is: Company *A* — 30%; Company *B* — 15%; and Company *C* — 55%.

(B) On the average, it will take 5 years for a department to decide to use a calculator from one of these companies in their courses.

REVIEW EXERCISE, page 391

22.

$$P = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ .2 & .6 & .2 \end{bmatrix} \end{array}$$

(A) $S_0 = [\,0 \quad 0 \quad 1\,]$

(B) $S_0 = [\,.5 \quad .3 \quad .2\,]$

23.

$$P = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ .2 & .6 & .2 \end{bmatrix} \end{array}$$

(A) $S_0 = [\,0 \quad 0 \quad 1\,]$

(B) $S_0 = [\,.5 \quad .3 \quad .2\,]$

REVIEW EXERCISE, page 392

The following transition matrix P is proposed as a model for the data, where I represents the population of Internet users.

Five years later

$$\begin{array}{cc} & \\ \text{Current} & I \\ \text{Year} & I' \end{array} \begin{array}{c} \begin{array}{cc} I & I' \end{array} \\ \begin{bmatrix} .95 & .05 \\ .40 & .60 \end{bmatrix} \end{array} = P$$

(A) Let $S_0 = [\,.14 \quad .86\,]$, and find S_1 and S_2. (Compute both matrices exactly and then round entries to two decimal places.)

REVIEW EXERCISE, page 393

The following transition matrix P is proposed as a model for the data, where S represents the population of smokers.

Five years later

$$\begin{array}{cc} & \\ \text{Current} & S \\ \text{year} & S' \end{array} \begin{array}{c} \begin{array}{cc} S & S' \end{array} \\ \begin{bmatrix} .74 & .26 \\ .03 & .97 \end{bmatrix} \end{array} = P$$

(A) Let $S_0 = [\,.301 \quad .699\,]$, and find S_1 and S_2. (Compute the matrices exactly and then round entries to three decimal places.)

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Section 10 – 2, page 411

26.

(A) There exists a 2×2 nonstrictly determined matrix game M for which the optimal strategy for player R is mixed and the optimal strategy for player C is pure.

(B) There exists a 2×2 nonstrictly determined matrix game M for which the optimal strategy for player R is to choose either row with probability $\frac{1}{2}$ and the optimal strategy for player C is to choose either column with probability $\frac{1}{2}$.

Section 10 – 2, page 412

39.

(A) Let $p_2 = 1 - p_1$ and $p_2 = 1 - p_1$ and simplify (4) to show that

$$E(P, Q) = [Dp_1 - (d - c)]q_1 + (b - d)p_1 + d$$

where $D = (a + d) - (b + c)$.

(B) Show that if is chosen so that $Dp_1 - (d - c) = 0$, that is, $p_1 = \frac{d-c}{D}$, then $v = \frac{\mathrm{ad-bc}}{D}$, regardless of the value of p_1.

40.

(A) Let $p_2 = 1 - p_1$ and $p_2 = 1 - p_1$ and simplify (4) to show that

$$E(P, Q) = [Dq_1 - (d - b)]p_1 + (c - d)q_1 + d$$

where $D = (a + d) - (b + c)$.

(B) Show that if q_1 is chosen so that $Dq_1 - (d - b) = 0$, that is, $q_1 = \frac{d-b}{D}$, then $v = \frac{\mathrm{ad-bc}}{D}$ regardless of the value of p_1.

Section 10 – 3, page 417

Step 2. Set up the two corresponding linear programming problems:

(A)

$$\begin{array}{lrcl} \text{Minimize} & y = x_1 + x_2 & & \\ \text{subject to} & ex_1 + gx_2 & \geq & 1 \\ & fx_1 + hx_2 & \geq & 1 \\ & x_1, x_2 & \geq & 0 \end{array}$$

(B)

$$\begin{array}{lrcl} \text{Minimize} & y = z_1 + z_2 & & \\ \text{subject to} & ez_1 + fz_2 & \geq & 1 \\ & gz_1 + hz_2 & \geq & 1 \\ & z_1, z_2 & \geq & 0 \end{array}$$

(B)

$$\begin{array}{lrcl} \text{Maximize} & y = z_1 + z_2 & & \\ \text{subject to} & ez_1 + fz_2 & \leq & 1 \\ & gz_1 + hz_2 & \leq & 1 \\ & z_1, z_2 & \geq & 0 \end{array}$$

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Section 10 – 3, page 418

Step 2. Set up the two corresponding linear programming problems:

(A)

$$\begin{array}{lrrcl} \text{Minimize} & y = x_1 + x_2 & & & \\ \text{subject to} & 2x_1 & +5x_2 & \geq & 1 \\ & 8x_1 & +x_2 & \geq & 1 \\ & & x_1, x_2 & \geq & 0 \end{array}$$

(B)

$$\begin{array}{lrrcl} \text{Minimize} & y = z_1 + z_2 & & & \\ \text{subject to} & 2z_1 & +5z_2 & \geq & 1 \\ & 8z_1 & +z_2 & \geq & 1 \\ & & z_1, z_2 & \geq & 0 \end{array}$$

(B)

$$\begin{array}{lrcl} \text{Maximize} & y = z_1 + z_2 & & \\ \text{subject to} & 2z_1 + 8z_2 & \leq & 1 \\ & 5z_1 + z_2 & \leq & 1 \\ & z_1, z_2 & \geq & 0 \end{array}$$

Step 4. Use the solutions in step 3 to find the value v_1 for the game M_1 and the optimal strategies and value for the original game M:

(A)

$$\begin{aligned} v_1 &= \frac{1}{x_1 + x_2} = \frac{1}{\frac{2}{19} + \frac{3}{19}} = \frac{19}{5} \\ p_1 &= v_1 x_1 = \frac{19}{5} \cdot \frac{2}{19} = \frac{2}{5} \\ p_2 &= v_1 x_2 = \frac{19}{5} \cdot \frac{3}{19} = \frac{3}{5} \end{aligned}$$

(B)

$$\begin{aligned} v_1 &= \frac{1}{z_1 + z_2} = \frac{1}{\frac{7}{38} + \frac{3}{38}} = \frac{19}{5} \\ q_1 &= v_1 z_1 = \frac{19}{5} \cdot \frac{7}{38} = \frac{7}{10} \\ q_2 &= v_1 z_2 = \frac{19}{5} \cdot \frac{3}{38} = \frac{3}{10} \end{aligned}$$

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Section 10 – 4, page 420

Step 2. Set up the two linear programming problems (the maximization problem is always the dual of the minimization problem):

(A)

Minimize $y = x_1 + x_2$

subject to

$$\begin{aligned} a_1 x_1 + b_1 x_2 &\geq 1 \\ a_2 x_1 + b_2 x_2 &\geq 1 \\ a_3 x_1 + b_3 x_2 &\geq 1 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(B)

Maximize $y = z_1 + z_2 + z_3$

subject to

$$\begin{aligned} a_1 z_1 + a_2 z_2 + a_3 z_3 &\boxed{\geq 1} \quad \leq 1 \\ b_1 z_1 + b_2 z_2 + b_3 z_3 &\boxed{\geq 1} \quad \leq 1 \\ z_1, z_2, z_3 &\geq 0 \end{aligned}$$

Section 10 – 4, page 421

Step 2. Set up the two corresponding linear programming problems:

(A)

Minimize $y = x^1 + x^2$ → $y = x_1 + x_2$

subject to

$$\begin{aligned} 5x_1 + 3x_2 &\geq 1 \\ 3x_1 + 6x_2 &\geq 1 \\ 10x_1 + x_2 &\geq 1 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(B)

Maximize $y = z^1 + z^2 + z^3$ → $y = z_1 + z_2 + z_3$

subject to

$$\begin{aligned} 5z_1 + 3z_2 + 10z_3 &\leq 1 \\ 3z_1 + 6z_2 + z_2 &\leq 1 \\ z_1, z_2 z_3 &\geq 0 \end{aligned}$$

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Section 10 – 4, page 423

Step 5. A further check is provided by showing that

$$P^*MQ^* = v$$

This we leave to the reader.

Conclusion: If the investor splits the \$10,000 proportional to the numbers in his optimal strategy, \$6,000 $\left(\frac{3}{5} \text{ of} \$10{,}000\right)$ in bonds and \$4,000 $\left(\frac{2}{5} \text{ of} \$10{,}000\right)$ in gold, then no matter which strategy fate chooses for interest rate changes (as long as the payoff matrix remains unchanged), the investor will be guaranteed a return of \$200 $\boxed{\left(\frac{2}{5} \text{ of} \$10{,}000\right)}$ If fate plays other than the optimal column strategy, the investor can do no worse than a \$200 return, and may do quite a bit better.

$\left(\frac{1}{5} \text{ of } \$1{,}000\right)$

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MATH 209: Corrections to eText

Maria Torres, 2018

The Textbook has errata and it is included here.

eTextbook

Section 1-3

Page 29

The phrase

we also will learn how to analyze a linear model based on real-word data.

Has to be changed to

we also will learn how to analyze a linear model based on real-world data.

Example 4 part (B)

The last phrase

So approximately 32,000,00 tons of carbon monoxide will be emitted in 2010.

must change to

So approximately 32,000,000 tons of carbon monoxide will be emitted in 2010.

Section 2-1

Page 55

The Revenue function is incorrect

R = (numdber of items sold) + (price per item) = $xp = x(mnx)$

Must be changed to product not sum

R = (number of items sold)(price per item) = $xp = x(mnx)$

Section 2-2

Page 66

Example 2 part (B)

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How are the graphs of $y = |x| + 4$ and $y = |x - 5|$ related to the graph of $y = |x|$? Confirm your answer by graphing all three functions simultaneously in the same coordinate system.

Must change to

How are the graphs of $y = |x + 4|$ and $y = |x - 5|$ related to the graph of $y = |x|$? Confirm your answer by graphing all three functions simultaneously in the same coordinate system.

In the solution

(B) The graph of $y = |x| + 4$ is the same as the graph of $y = |x|$ shifted to the left 4 units

Must to be changed to

(B) The graph of $y = |x + 4|$ is the same as the graph of $y = |x|$ shifted to the left 4 units

Section 2-3

Page 90

Exercises 21 and 22 are both labeled as 21

Page 91

33. Explain under what graphical conditions a quadratic function has exactly one real zero.

34 Explain under what graphical conditions a quadratic function has no real zeros.

Must be changed to

33. Explain under what graphical conditions a quadratic function has exactly one x-intercept.

34 Explain under what graphical conditions a quadratic function has no x-intercepts.

The concept of real zero is not defined in the textbook.

Chapter 2 Review

Page 119

Fourth point

If in an equation in two variables we get exactly one ouput for each input

Must be changed to

If in an equation in two variables we get exactly one output for each input

Sixth point

The functions specified by equations of the form $y = mx + b$, where are called

Must be changed to

The functions specified by equations of the form $y = mx + b$, where $m \neq 0$, are called

Page 120

The inverse of the exponential function with base b is called the logarithmic function with base b, denoted The domain

Must be changed to

The inverse of the exponential function with base b is called the logarithmic function with base b, denoted $\log_b x$. The domain

Section 3-3

Page 152

Step (3)

$1.30S - S = 100(1.03)^6 100$ Notice how many terms drop out .

Must be changed to

$1.03S - S = 100(1.03)^6 100$ Notice how many terms drop out .

Solution Manual

Section 1-2

Exercise 47 has 2 parts, but the answer has 3.

Part (c) must be deleted.

Section 2-5

Page S-63

Label of Exercise 2-2 must be changed to Exercise 2-5

MATH 209: Corrections to eText

Sam Fefferman, 2020

Matched problem 6 page 167: Monthly payment is \$518.32 with \$3000 rebate

Matched Problem 5 page 206 Solution (etext): Let x4=s and x5 =t

X1 =0.5-0.5s +1.5t

X2 = -5 -6s +6t

X3 = -1.5 -2.5s +2.5t

Note that the correction posted previously re Matched Problem 5 page 206 solution was based on the hard copy

version of the text, where the original question is slightly different than the etext version

PROCEDURE

2 × 2 Matrix Games and Linear Programming: Geometric Approach

Given the nonstrictly determined matrix game

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

to find $P^* = [p_1 \quad p_2]$, $Q^* = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$, and v, proceed as follows:

Step 1. If M is not a positive matrix (one with all entries positive), convert it into a positive matrix M_1 by adding a suitable positive constant k to each element. Let M_1, the new positive matrix, be represented as follows:

$$M_1 = \begin{bmatrix} e & f \\ g & h \end{bmatrix} \qquad \begin{matrix} e = a + k & f = b + k \\ g = c + k & h = d + k \end{matrix}$$

This new matrix game, M_1, has the same optimal strategies P^* and Q^* as M. However, if v_1 is the value of the game M_1, then

corrected

$$v = v_1 - k$$

is the value of the original game M.

Step 2. Set up the two corresponding linear programming problems:

(A) Minimize $y = x_1 + x_2$
subject to $ex_1 + gx_2 \geq 1$
$fx_1 + hx_2 \geq 1$
$x_1, x_2 \geq 0$

(B) Maximize $y = z_1 + z_2$
subject to $ez_1 + fz_2 \leq 1$
$gz_1 + hz_2 \leq 1$
$z_1, z_2 \geq 0$

Step 3. Solve each linear programming problem geometrically.

Step 4. Use the solutions in step 3 to find the value v_1 for game M_1 and the optimal strategies and value v for the original game M:

$$v_1 = \frac{1}{y} = \frac{1}{x_1 + x_2} \quad \text{or} \quad v_1 = \frac{1}{y} = \frac{1}{z_1 + z_2}$$

$$P^* = [p_1 \quad p_2] = [v_1x_1 \quad v_1x_2] \qquad Q^* = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} v_1z_1 \\ v_1z_2 \end{bmatrix}$$

$$v = v_1 - k$$

Step 5. A further check of the solution is provided by showing that

$$P^*MQ^* = v \qquad \text{See Theorem 3, Section 10-2.}$$

INSIGHT

In step 2 in the box, note that part (B) is the dual of part (A), and consequently, both must have the same optimal value (Theorem 1, Section 6-3). In the next section, using the simplex method and properties of the dual, we will see that solving part (B) will automatically produce the solution for part (A). In this section we restrict our attention to the geometric approach; hence, we must solve each part as a separate problem.

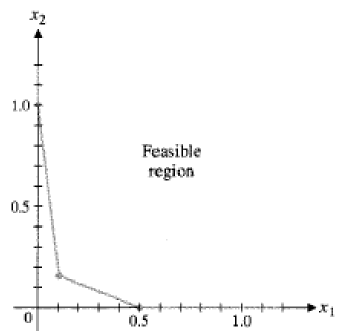
corrected

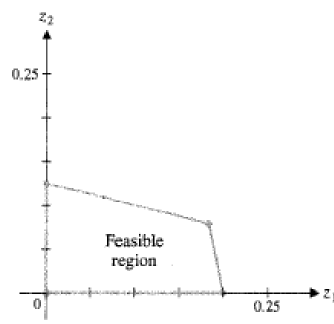
Step 2. Set up the two corresponding linear programming problems:

(A) Minimize $y = x_1 + x_2$
subject to
$$2x_1 + 5x_2 \ge 1$$
$$8x_1 + x_2 \ge 1$$
$$x_1, x_2 \ge 0$$

(B) Maximize $y = z_1 + z_2$
subject to
$$2z_1 + 8z_2 \le 1$$
$$5z_1 + z_2 \le 1$$
$$z_1, z_2 \ge 0$$

Step 3. Solve each linear programming problem geometrically:





Theorems 1 and 2 in Section 5-3 imply that each problem has a solution that must occur at a corner point.

(A)

Corner Points	Minimize $y = x_1 + x_2$
$(0, 1)$	1
$(\frac{2}{19}, \frac{3}{19})$	$\frac{5}{19}$
$(\frac{1}{2}, 0)$	$\frac{1}{2}$

Min y occurs at

$x_1 = \frac{2}{19}$ and $x_2 = \frac{3}{19}$

(B)

Corner Points	Maximize $y = z_1 + z_2$
$(0, 0)$	0
$(0, \frac{1}{8})$	$\frac{1}{8}$
$(\frac{7}{38}, \frac{3}{38})$	$\frac{5}{19}$
$(\frac{1}{5}, 0)$	$\frac{1}{5}$

Max y occurs at

$z_1 = \frac{7}{38}$ and $z_2 = \frac{3}{38}$

Step 4. Use the solutions in step 3 to find the value v_1 for the game M_1 and the optimal strategies and value v for the original game M:

(A) $$v_1 = \frac{1}{x_1 + x_2} = \frac{1}{\frac{2}{19} + \frac{3}{19}} = \frac{19}{5}$$

$$p_1 = v_1 x_1 = \frac{19}{5} \cdot \frac{2}{19} = \frac{2}{5}$$
$$p_2 = v_1 x_2 = \frac{19}{5} \cdot \frac{3}{19} = \frac{3}{5}$$

(B) $$v_1 = \frac{1}{z_1 + z_2} = \frac{1}{\frac{7}{38} + \frac{3}{38}} = \frac{19}{5}$$

$$q_1 = v_1 z_1 = \frac{19}{5} \cdot \frac{7}{38} = \frac{7}{10}$$
$$q_2 = v_1 z_2 = \frac{19}{5} \cdot \frac{3}{38} = \frac{3}{10}$$

Note: v_1 found in part (A) should always be the same as v_1 found in part (B).

Optimal strategies are the same for both games M and M_1. Thus,

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